

المحاضرة ... (١)

ملاحظات مختلفة

$\int \vec{A}\, d\vec{l}$

$(1,1,1)$ ، $(0,0,0)$

$\vec{l} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$

$\vec{A} = \vec{i}xy + \vec{k}xyz - \vec{j}z^2$ ... $\vec{A}$

$\vec{l} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$

$d\vec{l} = dt\,\vec{i} + 2t\,dt\,\vec{j} + 3t^2 dt\,\vec{k}$

$\vec{A} = xy\,\vec{i} - z^2\vec{j} + xyz\,\vec{k}$

t^2, y, x, t ... x ... $\vec{A}$

t^3 ... z

$\vec{A} = t^3\vec{i} - t^6\vec{j} + t^6\vec{k}$

$\Rightarrow \vec{A}\, d\vec{l} = t^3 dt - 2t^7 dt + 3t^8 dt$

$\Rightarrow \int \vec{A}\, d\vec{l} = \int_0^1 t^3 dt - 2\int_0^1 t^7 dt + 3\int_0^1 t^8 dt$

$= \left[\frac{t^4}{4} - \frac{2t^8}{8} + \frac{3t^9}{9}\right]_0^1$

$= \left[\frac{1}{4} - \frac{1}{4} + \frac{3}{9}\right] = \frac{1}{3}$

$\vec{A} = \vec{j}x + \vec{k}y$ ... $\vec{A}$ ...

... XY ...

...

$\int \vec{A}\, d\vec{l} = \int \text{rot}\vec{A}\, d\vec{s}$

...

$\vec{l} = x\vec{i} + y\vec{j}$

z ... xy ... a

$\vec{A} = x\vec{j} + y\vec{k}$

$d\vec{l} = dx\,\vec{i} + dy\,\vec{j}$

$\Rightarrow \vec{A}\, d\vec{l} = x\,dy$

$\int \vec{A}\, d\vec{l} = \int x\,dy = x\int_0^a dy$

$\int \vec{A}\, d\vec{l} = a[y]_0^a = a^2$

...

$$\vec{rot}\,\vec{A} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & y \end{vmatrix}$$

$\vec{ds} = dy\,dz\,\vec{i} + dx\,dz\,\vec{j} + dx\,dy\,\vec{k}$

...

لكن $dz = 0$ $\vec{ds} = dx\,dy\,\vec{k}$

$\iint [\vec{\nabla}\times\vec{A}]\,d\vec{s} = \iint dx\,dy$

$= [x]_0^a\,[y]_0^a = a^2$

...

أوجد ... $z = x^2 + y^2$ ... $(1,2,4)$

$x^2 + y^2 - z = 0$

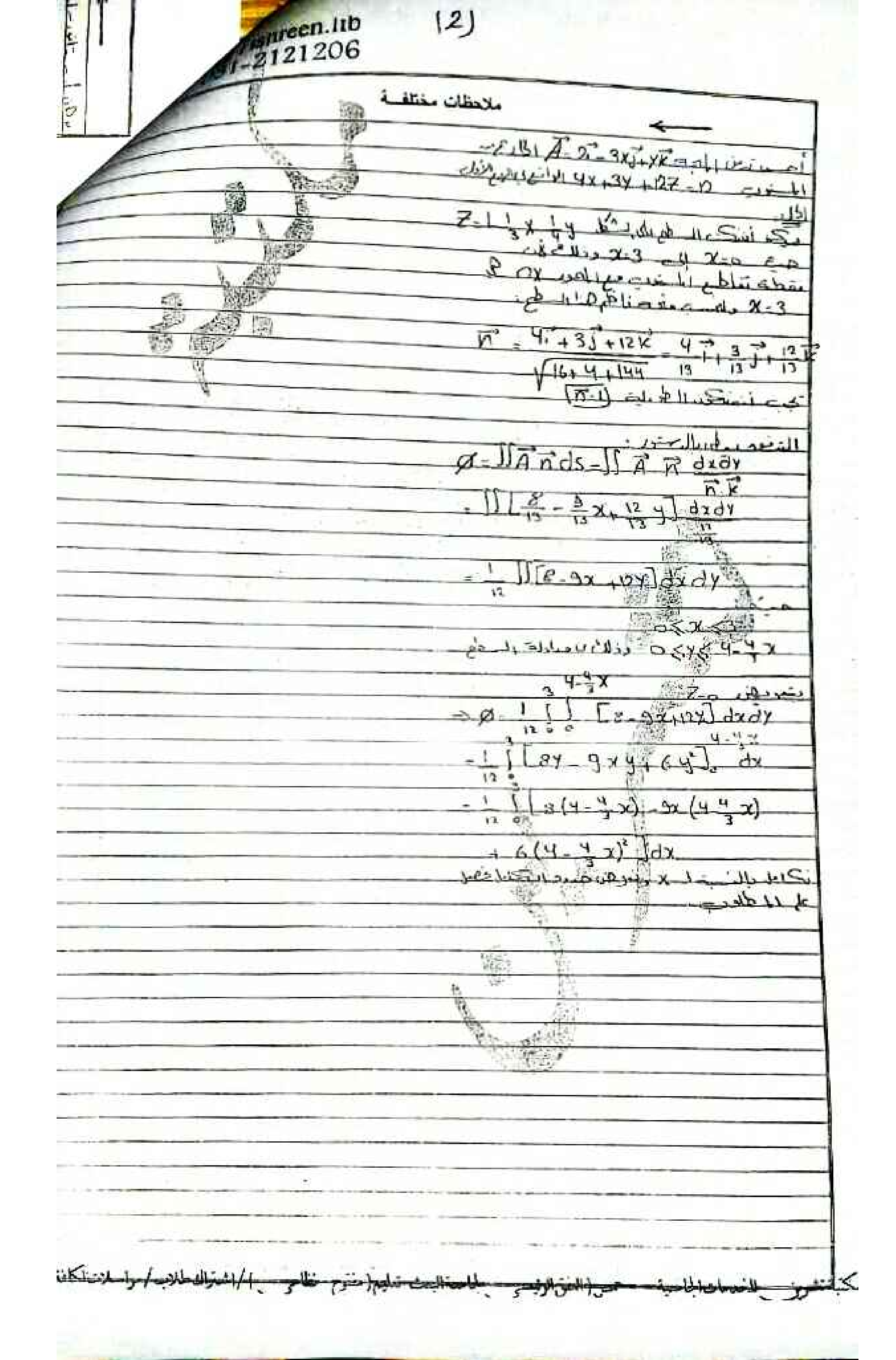
$$\vec{n} = \frac{\nabla\vec{f}}{|\nabla f|} = \frac{2x\vec{i} + 2y\vec{j} - \vec{k}}{\sqrt{4x^2 + 4y^2 + 1}}\Bigg|_{(1,2,4)}$$

$$= \frac{2\vec{i} - 4\vec{j} - \vec{k}}{\sqrt{21}} = \frac{2}{\sqrt{21}}\vec{i} - \frac{4}{\sqrt{21}}\vec{j} - \frac{1}{\sqrt{21}}\vec{k}$$

...

$\int \vec{A}\, d\vec{s} = \int \vec{\nabla}\cdot\vec{A}\, dt = \int \text{div}\,\vec{A}\, dt$

غوص $\int E\,ds = \frac{Q}{\varepsilon_0}$



ملاحظات مختلفة

أحسب تدفق الحقل المتجه $\vec{A} = z\vec{i} - 3x\vec{j} + y\vec{k}$ عبر الجزء من المستوي $4x + 3y + 12z = 12$ الواقع في الربع الأول

الحل:
يمكن أن نكتب المستوي بالشكل $z = 1 - \frac{1}{3}x - \frac{1}{4}y$
حيث من $x=0$ إلى $x=3$ وذلك لأن
نقطة تقاطع المستوي مع المحور ox هي
$x=3$ وهي نقطة من المستوي

$$\vec{n} = \frac{4\vec{i} + 3\vec{j} + 12\vec{k}}{\sqrt{16 + 9 + 144}} = \frac{4}{13}\vec{i} + \frac{3}{13}\vec{j} + \frac{12}{13}\vec{k}$$

يجب أن نتحقق أن $\boxed{|\vec{n}| = 1}$

التدفق يعطى بالعلاقة:

$$\phi = \iint \vec{A}\cdot\vec{n}\, ds = \iint \vec{A}\cdot\vec{n}\, \frac{dx\,dy}{\vec{n}\cdot\vec{k}}$$

$$= \iint \left[\frac{8}{13} - \frac{9}{13}x + \frac{12}{13}y\right] \frac{dx\,dy}{\frac{12}{13}}$$

$$= \frac{1}{12}\iint [8 - 9x + 12y]\, dx\,dy$$

حيث:
$0 \le x \le 3$
$0 \le y \le 4 - \frac{4}{3}x$ وذلك من معادلة المستوي
بتعويض $z=0$

$$\Rightarrow \phi = \frac{1}{12}\int_0^3 \int_0^{4-\frac{4}{3}x} [8 - 9x + 12y]\, dx\,dy$$

$$= \frac{1}{12}\int_0^3 \left[8y - 9xy + 6y^2\right]_0^{4-\frac{4}{3}x} dx$$

$$= \frac{1}{12}\int_0^3 \Big[8\left(4 - \frac{4}{3}x\right) - 9x\left(4 - \frac{4}{3}x\right) + 6\left(4 - \frac{4}{3}x\right)^2\Big] dx$$

نكامل بالنسبة لـ x ونعوض حدود التكامل فنحصل
على المطلوب

ملاحظات مختلفة

يمكن أن تتغير [illegible] الشحنة [illegible]

$$\nabla J = -\frac{\partial \rho}{\partial t}$$

لنفرض أن كثافة الشحنة الحجمية هي ρ وبالتالي [illegible]

$$Q = \int \rho \, d\tau$$

[illegible]

$$I = \frac{dQ}{dt}$$

[illegible]

$$I = -\int \frac{\partial \rho}{\partial t} \, d\tau$$

[illegible]

$$I = \int_S J \, ds$$

$$\int_S \vec{J} \cdot d\vec{s} = -\int_\tau \frac{\partial \rho}{\partial t} \, d\tau$$

[illegible]

$$\int \nabla J \, d\tau = -\int \frac{\partial \rho}{\partial t} \, d\tau$$

[illegible]

$$\nabla J = -\frac{\partial \rho}{\partial t}$$

معادلة الاستمرارية

[illegible]

[illegible] $(I\,dl)$ [illegible] (B) [illegible] $(I\,dl)$ [illegible] (R) [illegible] P [illegible]

$$dB = \frac{\mu_0}{4\pi} \frac{I \, dl}{r^2}$$

[illegible] dB [illegible] $(r, I\,dl)$ [illegible] $(I\,dl)$ [illegible] $(I\,dl)$ [illegible] dB' [illegible] (P) [illegible] (dB', dB_y) [illegible]

[illegible] (dB_x, dB_y) [illegible] (dB_x', dB_x) [illegible]

$$B = \int dB \sin\alpha$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\alpha}{r^2}$$

[illegible] $(I\,dl)$ [illegible] r [illegible] α [illegible]

$$B = \frac{\mu_0 I}{4\pi} \frac{\sin\alpha}{r^2} \int dl$$

$$\int dl = 2\pi R$$

$$B = \frac{\mu_0 I}{2} \frac{\sin\alpha}{r^2} R$$

[illegible]

$$\sin\alpha = \frac{R}{r} \; ; \; r^2 = R^2 + a^2$$

[illegible] r^2، $\sin\alpha$ [illegible]

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(a^2 + R^2)^{3/2}}$$

$$[P_x, H_x] = 0$$

$$[\hat{P}_x, \hat{H}_x] = [P_x H - H P_x]$$

[illegible]

$$\left[\left(-i\hbar \frac{\partial}{\partial x}\right)\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\right) - \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\right)\left(-i\hbar \frac{\partial}{\partial x}\right)\right]\psi(x) =$$

$$\left(-i\hbar \frac{\partial}{\partial x}\right)\left(\hbar^2 \frac{\partial^2}{\partial x^2}\psi(x)\right) - \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\right)\left(-i\hbar \frac{\partial}{\partial x}\psi(x)\right)$$

[illegible]

برهن أن [illegible] $V(x) = V(-x)$

$$\hat{H}\psi(x) = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x)$$

[illegible] $(\hat{\pi})$ [illegible]

$$\hat{\pi}\hat{H}\psi(x) = -\frac{\hbar^2}{2m} \hat{\pi} \frac{d^2\psi(x)}{dx^2} + \hat{\pi} V(x)\psi(x)$$

$$= -\frac{\hbar^2}{2m} \frac{d^2\psi(-x)}{dx^2} + V(x)\psi(-x)$$

$$= -\frac{\hbar^2}{2m} \frac{d^2\psi(-x)}{dx^2} + V(x)\psi(-x) = \hat{H}\hat{\pi}\psi(x)$$

$$\Rightarrow (\hat{\pi}\hat{H} - \hat{H}\hat{\pi})\psi(x) = 0 \Rightarrow [\hat{H}, \hat{\pi}] = 0$$

ملاحظات مختلفة

2012

برهن أن ... [illegible] ... العلاقة:

$$\frac{\mu}{D} = \frac{q}{kT}$$

الحل:

نفترض أن [illegible] ... غير [illegible] ... E ... [illegible]

ولنفرض أن q هي شحنة الأيون وأن الطاقة [illegible]

$$\mathcal{E}(x) = qE_x \quad (1)$$

* بشرط أن تركيز الشوارد غير كبير [illegible] ... تركيز الشوارد [illegible]

* [illegible] ... الأفعال [illegible]

$$\frac{n(x)}{n(0)} = e^{-\frac{\mathcal{E}(x)}{kT}} = e^{-\frac{qE_x}{kT}} \quad (2)$$

حيث [illegible] تركيز الشوارد عند $x = 0$

T ... [illegible]

[illegible] ... (μE) في اتجاه الحقل [illegible]

$$J_\mu = n(x)\,\mu E \quad (3)$$

وإذا كان [illegible] ... الشوارد [illegible] ... x ...

$$J_D = D\,\frac{dn(x)}{dx} \quad (4)$$

[illegible] ... في اتجاه تزايد التركيز [illegible]

$$J_\mu + J_D = 0$$

$$n(x)\,\mu E = D\,\frac{dn(x)}{dx} \quad (5)$$

لنفاضل $n(x)$ [illegible]

$$\frac{dn(x)}{dx} = n(0)\,\frac{qE}{kT}\,e^{\frac{qE_x}{kT}} \quad (6)$$

نعوض هذه القيمة في العلاقة الأخيرة [illegible]

$$n(x)\,\mu E = D\,n(0)\,\frac{qE}{kT}\,e^{\frac{qE_x}{kT}} \quad (7)$$

[illegible] $n(x) = n(0)\,e^{\frac{qE_x}{kT}}$ [illegible]

$$\mu E = D\,\frac{qE}{kT} \Rightarrow \mu = \frac{Dq}{kT}$$

أي أن:

$$\frac{\mu}{D} = \frac{q}{kT}$$

وهو المطلوب.

برهن أن الدالات الخاصة [illegible] ... هي دالات متعامدة [illegible] ... Eigenfunctions [illegible]

الحل:

لنفرض أن الدالتين U_n و U_m [illegible] ... المؤثر هرميتي $\hat{A}$.

$$\hat{A}U_n = a_n U_n \quad \text{الأولى}$$

$$\hat{A}U_m = a_m U_m \quad \text{الثانية} \qquad a_n \neq a_m$$

نضرب المعادلة الأولى بالدالة U_m^* [illegible] ... على كل المجال:

$$\int_{-\infty}^{+\infty} U_m^* \hat{A} U_n\,dx = a_n \int_{-\infty}^{+\infty} U_m^* U_n\,dx$$

[illegible] ... المعادلة الثانية [illegible]

$$\hat{A}^* U_m^* = a_m^* U_m^* = a_m U_m^*$$

[illegible] ... a_m [illegible] ... المؤثر هرميتي [illegible] ... المعادلة الأخيرة [illegible]

$$\int_{-\infty}^{+\infty} \hat{A}^* U_m^* U_n\,dx = a_m \int_{-\infty}^{+\infty} U_m^* U_n\,dx$$

[illegible] ... $\hat{A}$ مؤثر هرميتي [illegible] ... المعادلة [illegible]

$$a_m \int_{-\infty}^{+\infty} U_m^* U_n\,dx = a_n \int_{-\infty}^{+\infty} U_m^* U_n\,dx$$

أي:

$$(a_m - a_n) \int_{-\infty}^{+\infty} U_m^* U_n\,dx = 0$$

وبما أن $a_m \neq a_n$ فإن:

$$\int_{-\infty}^{+\infty} U_m^* U_n\,dx = 0$$

أي أن الدالتين U_m و U_n متعامدتان [illegible]

[illegible] ... $[x, a/2]$ [illegible]

$$\psi(x) = \left(\frac{2}{a}\right)^{\frac{1}{2}} \sin\frac{2\pi x}{a}$$

$$\int_0^{\frac{a}{4}} \psi\,\psi^*\,dx = \int_0^{\frac{a}{4}} \psi^2\,dx = \int_0^{\frac{a}{4}} \left(\frac{2}{a}\right)^{\frac{1}{2}} \sin\frac{2\pi x}{a} \left(\frac{2}{a}\right)^{\frac{1}{2}} \sin\frac{2\pi x}{a}$$

$$= \frac{2}{a} \int_0^{\frac{a}{4}} \left(\sin\frac{2\pi x}{a}\right)^2 dx =$$

$$= \frac{2}{a}\left[\frac{x}{2} - \frac{1}{4}\,\frac{a}{2\pi}\sin 2\,\frac{2\pi x}{a}\right]_0^{\frac{a}{4}}$$

$$\left(b = \frac{2\pi}{a}\right) \int \sin^2 bx\,dx = \frac{x}{2} - \frac{1}{4b}\sin(2bx)$$

$$= \frac{2}{a}\left[\frac{a}{8} - \frac{1}{4}\,\frac{a}{2\pi}\sin\frac{4\pi a}{4a} - 0\right] = \frac{2}{a}\,\frac{a}{8} = \frac{1}{4}$$

ملاحظات مختلفة ← $\vec{n} = \frac{\nabla\phi}{|\nabla\phi|}$

أوجد متجه الواحدة العمودي على سطح $z = 3x^2 + 2y^2$ في النقطة $(1, 2, 4)$ ؟

الحل:

$$\phi(x, y, z) = 3x^2 + 2y^2 - z$$

[illegible] العمودي على السطح [illegible] بالشكل التالي:

$$\vec{\nabla}\phi = 6x\vec{i} + 4y\vec{j} - \vec{k}$$

[illegible] العمودي على السطح في النقطة $(1, 2, 4)$ هو:

$$\vec{\nabla}\phi = 6\vec{i} + 8\vec{j} - 4\vec{k}$$

[illegible] هي:

$$|\vec{\nabla}\phi| = \sqrt{36 + 64 + 16} = \sqrt{116} = 10.8$$

[illegible] العمودي [illegible] هو:

$$\vec{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|} = \frac{6}{10.8}\vec{i} + \frac{8}{10.8}\vec{j} - \frac{4}{10.8}\vec{k}$$

R

[illegible] الثابتة [illegible]

dq [illegible] (σ)

الحل:

[illegible]

[illegible] (x) [illegible]

$$dq = \sigma\, ds = \sigma\, 2\pi x\, dx$$

[illegible] بالعلاقة التالية:

$$dE' = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u} = \frac{\sigma}{2\varepsilon_0}\frac{x\,dx}{r^2}\vec{u}$$

[illegible] dE [illegible]

[illegible]

$$dE = dE'\cos\alpha$$

$$= \frac{\sigma}{2\varepsilon_0}\, x\,\frac{dx}{r^2}\cos\alpha$$

وبالتالي:

$$E = \int dE = \frac{\sigma}{2\varepsilon_0}\int\frac{x\,dx}{r^2}\cos\alpha$$

[illegible]

$$x = z\,\tan\alpha ; \quad dx = \frac{z}{\cos^2\alpha}\,d\alpha$$

$$r = \frac{z}{\cos\alpha}$$

نعوض:

$$E = \frac{\sigma}{2\varepsilon_0}\int_0^{\theta}\frac{z^2\tan\alpha\cdot\cos^2\alpha}{\cos^2\alpha\cdot z^2}\cos\alpha\, d\alpha$$

$$E = \frac{\sigma}{2\varepsilon_0}\int_0^{\theta}\sin\alpha\, d\alpha$$

وبالتالي [illegible] :

$$E = \frac{\sigma}{2\varepsilon_0}(1 - \cos\theta)$$

أي:

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right)$$

[illegible]

$$\varepsilon_0 = 8.85\times10^{-12}\ F\, m^{-1}$$

[illegible]

$$F = \frac{q_1 q_2}{r^2} = \frac{3\times10^{3}\times3\times10^{3}}{(100)^2} = 9\times10^{14}\ dyn$$

$$F = 9\times10^{9}\ N$$

[illegible]

$$F = 9\times10^{9} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2} = \frac{1\times1}{4\pi\times9\times10^{9}\times(1)^2} = 8.85\times10^{-12}$$

[illegible]:

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

$$\varepsilon_0 = \frac{Coul^2}{N m^2} = \frac{Coul^2}{Joul\cdot m} = \frac{Coul}{Volt\cdot m}$$

$$= \frac{Farad}{m}$$

ملاحظات مختلفة ←

برهن أن الدالة التالية:

$$\phi(x,y,z) = \sin 2x \sin 3y \sin 4z$$

هي دالة ذاتية للمؤثر

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

الحل

$$\frac{\partial^2}{\partial x^2}\phi(x,y,z) = -4\sin 2x \sin 3y \sin 4z$$

$$= -4\,\phi(x,y,z)$$

$$\frac{d^2}{dy^2}\phi(x,y,z) = -9\sin 2x \sin 3y \sin 4z$$

$$= -9\,\phi(x,y,z)$$

$$\frac{d^2}{dz^2}\phi(x,y,z) = -16\sin 2x \sin 3y \sin 4z$$

$$= -16\,\phi(x,y,z)$$

وبجمع هذه العلاقات نجد:

$$\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)\phi(x,y,z)$$

$$= -29\,\phi(x,y,z)$$

أي أن الدالة ذاتية وقيمتها الذاتية (-29)

استخدم نظرية ... لحساب الشحنة الكلية ... مكعب طول ضلعه (2cm) ... في حقل كهربائي

$$\vec{E} = 2ax^2\vec{i}$$

الحل

نفرض q هي الشحنة ...

$$\oint_S E\,ds = \frac{q}{\varepsilon_0}$$

$$\oint_S E\,ds = \int \nabla E\, d\tau = \frac{q}{\varepsilon_0}$$

$$\nabla E = \frac{\partial E_x}{\partial x} = 4ax$$

$$\int \nabla \vec{E}\, d\tau = \int_\tau 4ax\, d\tau = 4a\int_0^2 x\,dx \int_0^2 dy \int_0^2 dz$$

$$= 16a\left[\frac{x^2}{2}\right]_0^2 = 32a \Rightarrow$$

$$q = 32\varepsilon_0 a$$

برهن أن تفرق ... كثافة الحقل المغناطيسي ... معدوم

الحل

لنعبر عن كثافة الحقل المغناطيسي $\vec{B}$ بواسطة ... ونستخدم ... على النحو التالي:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int d\vec{l} \times \left(-\nabla\frac{1}{r}\right)$$

لأن ... بالمكعب ...

$$\frac{\vec{r}}{r^3} = -\nabla\left(\frac{1}{r}\right)$$

وبالتالي نجد:

$$\vec{B} = \frac{\mu_0}{4\pi}\nabla \times \int \frac{I\,d\vec{l}}{r}$$

نأخذ ...

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}}{r}$$

...

$$\vec{B} = \nabla \times \vec{A}$$

... $\nabla\cdot(\nabla \times \vec{A}) = 0$ ...

$$\nabla\cdot\vec{B} = 0$$

... $\nabla \times \vec{B} = \mu_0 \vec{J}$ ...

$$\oint_C \vec{B}\,d\vec{l} = \int_S \nabla \times \vec{B}\, ds = \int_S \mu_0 \vec{J}\, ds$$

ومنه:

$$I = \int_S J\,ds \Rightarrow \oint_C B\,dl = \mu_0 I$$

(7)

ملاحظات مختلفة

$\text{div}\, \text{rot}\, \vec{A} = 0$

$\text{div}(\text{rot}\,\vec{A}) = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})$

[illegible]

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

[illegible]

$\bar{n} = \frac{1}{e^{\frac{\varepsilon-\mu}{KT}} - 1}$

[illegible]

$Y = U - TS - \mu \bar{N}$

[illegible]

$P = \left[\frac{\partial (PV)}{\partial V}\right]_{T\mu}, \quad S = \left[\frac{\partial (PV)}{\partial T}\right]_{V\mu}$

$N = \left[\frac{\partial (PV)}{\partial \mu}\right]_{TV}$

$W(\varepsilon, n) = \Omega(\varepsilon, n)\, e^{\frac{\mu n - \varepsilon}{KT}}$

[illegible]

$Y = U - ST - \mu \bar{N} = -PV$

$U = E$ ، $PV = ST + \mu \bar{N} - E$

[illegible]

$\Omega(\varepsilon, n) = \Omega(n) = 1$

$d(PV) = SdT + PdV + Nd\mu$

$dU = TdS - PdV + \mu d\bar{N}$

$W(\varepsilon, n) = W(n) = e^{\frac{\mu - \varepsilon}{KT} n}$

$d(PV) = (PdV + VdP)$

$= TdS + SdT + \mu dN + N d\mu - TdS + PdV - \mu d\bar{N}$

$\sum_n e^{\frac{\mu - \varepsilon}{KT} n}$

$\bar{n} = KT \frac{\partial}{\partial \mu} \ln \sum_n e^{\frac{\mu - \varepsilon}{KT} n}$

$P = \left[\frac{\partial (PV)}{\partial V}\right]_{T\mu}, \quad S = \left[\frac{\partial (PV)}{\partial T}\right]_{V\mu}$

$\bar{n} = KT \frac{\partial}{\partial \mu} \ln \sum_n x^n$

$N = \left[\frac{\partial (PV)}{\partial \mu}\right]_{TV}$

[illegible]

$[\hat{E}, \hat{t}]\psi = \hat{t}\hat{E}\psi - \hat{E}\hat{t}\psi$

$i\hbar \frac{\partial}{\partial t} t\psi - t\left(i\hbar \frac{\partial}{\partial t}\right)\psi$

$i\hbar\left[t \frac{\partial \psi}{\partial t} + \psi\right] - i\hbar t \frac{\partial \psi}{\partial t} = i\hbar\psi \Rightarrow [\hat{E}, \hat{t}] = i\hbar$

ملاحظات مختلفة ←

[illegible]

[illegible]
ε_0

$$\oint \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0}$$

[illegible]

$$\vec{E} = K \frac{Q}{r^2} \vec{u}_r$$

[illegible] (ds) [illegible]

$$\vec{E}\, d\vec{s} = K Q \frac{\vec{u}_r\, d\vec{s}}{r^2}$$

[illegible] $\vec{u}_r \cdot d\vec{s}$ [illegible] $\vec{E}$ [illegible] $ds' = ds \cos\theta$ [illegible]

$$\vec{E}\, d\vec{s} = K Q \frac{ds'}{r^2}$$

[illegible] $\frac{ds'}{r^2}$ [illegible] $d\Omega$ [illegible] ds' [illegible]

$$\vec{E}\, d\vec{s} = K Q\, d\Omega$$

$$\oint \vec{E}\, d\vec{s} = \int K Q\, d\Omega = K Q \oint d\Omega$$

[illegible] (4π)

$$\oint d\Omega = 4\pi \quad , \quad K = \frac{1}{4\pi\varepsilon_0}$$

$$\oint \vec{E}\, d\vec{s} = \frac{1}{4\pi\varepsilon_0} Q\, 4\pi = \frac{Q}{\varepsilon_0}$$

[illegible]

$$\oint \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0}$$

[illegible]

$$\oint \vec{E}\, d\vec{s} = \nabla \vec{E}$$

$$\oint \vec{E}\, d\vec{s} = \int_\tau \nabla \vec{E}\, d\tau$$

[illegible]

$$\frac{Q}{\varepsilon_0} = \int_\tau \frac{\rho\, d\tau}{\varepsilon_0}$$

[illegible]

$$\int_\tau \nabla \vec{E}\, d\tau = \int_\tau \frac{\rho\, d\tau}{\varepsilon_0}$$

[illegible]

$$\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$$

[illegible]

[illegible] $P(r,\theta)$ [illegible]

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_2} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1}$$

$$\phi = \frac{Q}{4\pi\varepsilon_0 r_1 r_2}(r_1 - r_2)$$

[illegible]

$$r_1^2 = r_2^2 + l^2 + 2 r_2 l \cos\theta'$$

$$r_1^2 - r_2^2 = l^2 + 2 r_2 l \cos\theta'$$

$$r_1 - r_2 = \frac{l(l + 2r\cos\theta')}{r_1 + r_2}$$

[illegible]

$$\phi = \frac{Q l (l + 2 r_2 \cos\theta)}{4\pi\varepsilon_0 r_1 r_2 (r_1 + r_2)}$$

[illegible] $r \gg l$

$$r_1 = r_2 = r \qquad \theta = \theta'$$

$$\phi = \frac{Q l \cos\theta}{4\pi\varepsilon_0 r^2} \Rightarrow P = Q l$$

$$\Rightarrow \phi = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}$$

[illegible] $P = Q l$ [illegible]

$$\phi = \frac{\vec{P}\, \vec{u}_r}{4\pi\varepsilon_0 r^2}$$

[illegible]

$$U = \phi(\phi + \Delta\phi) \quad Q\phi - Q\Delta\phi$$

[illegible] $\Delta\vec{P}$ [illegible]

$$\Delta\phi = -\vec{E}\,\Delta\vec{P}$$

$$U = -\vec{E} Q \Delta\vec{P} = -\vec{E}\vec{P} = -E P \cos\theta$$





ملاحظات مختلفة

$dc = \oint_l \vec{A}\, d\vec{l} = \iint_s \text{rot}\,\vec{A}\, d\vec{s}$

$div\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\Delta q_i\, \Delta p_i \geq \hbar$ — علاقة هايزنبرغ

$S = K \ln W$ — مبدأ بولتزمان

$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \hat{V}(x)$ — المؤثر الهاملتوني

$\hat{P}_x = i\hbar \frac{\partial}{\partial x}$ — مؤثر الاندفاع الخطي

$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{u}$ — قانون كولوم

$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ — مؤثر لابلاس

$\phi = \oint_s \vec{E}\, d\vec{s} = \frac{1}{\varepsilon_0}\sum q_i + \frac{1}{2\varepsilon_0}\sum q_s$ — نظرية غاوس

$P_i = \frac{e^{-E_i/kT}}{\sum e^{-E_i/kT}} \Rightarrow P_i = \frac{e^{-E_i/kT}}{Z}$ — توزيع بولتزمان

$P_i = e^{\frac{F-E_i}{kT}}$

$\hat{\Pi}\,\psi(x) = \psi(-x)$ — مؤثر الانعكاس

$\int_{-\infty}^{\infty} \psi_1^* \hat{A}\, \psi_2\, dx = \int_{-\infty}^{\infty} (\hat{A}\psi_1)^* \psi_2\, dx$ — المؤثر الهرميتي

$\bar{X} = \sum x_i w_i$

$\bar{X} = \frac{\int_{x_1}^{x_2} x_i\, \rho(x)\, dx}{\int_{x_1}^{x_2} \rho(x)\, dx}$

[illegible]

(q_i, p_i) [illegible]

[illegible]

$dc = \vec{A}\, d\vec{l}$ [illegible]

[illegible]

$S = K \ln W$ [illegible]

المؤثر الهرميتي: [illegible]

$\int \psi_1^* \hat{A}\, \psi_2\, dx = \int (\hat{A}\psi_1)^* \psi_2\, dx$

[illegible]



ملاحظات مختلفة

[illegible]

$$F = ma$$

[illegible]

$$ma = -\alpha q \Rightarrow m x'' + \alpha x = 0$$

$$\Rightarrow x'' + \frac{\alpha}{m} x = 0$$

[illegible] : Const

[illegible]

$$\omega = \sqrt{\frac{\alpha}{m}}$$

$$x = A \sin(\omega t + \alpha)$$

$$P = m\omega A \cos(\omega t + \alpha)$$

$$\omega = \sqrt{\frac{\alpha}{m}} = 2\pi\nu$$

[illegible] (P, X)

$$\left(\frac{q'}{A}\right)^2 + \left(\frac{P}{m\omega A}\right)^2$$

[illegible] $m\omega A$ [illegible]

$$S = \oint p\,dq = \pi\, ab = \pi\, m\omega A^2$$

[illegible]

$$E = \frac{P^2}{2m} + \frac{1}{2}\alpha a^2 = \frac{\alpha}{2} A^2$$

[illegible]

$$E = \frac{S\alpha}{2\pi m\omega} = \alpha \oint p\,dq$$

$$E = \frac{\alpha}{2}\left(\frac{S}{\pi m\omega}\right) = \frac{\alpha \oint p\,dp}{2\pi m\omega}$$

$$= \frac{m\omega^2 \oint p\,dq}{2\pi m\omega}$$

$$E = \frac{\omega \oint p\,dq}{2\pi} = \nu \oint p\,dq$$

[illegible]

$$dB = \frac{\mu_0 I r^2 \sin\alpha\, N}{2 r^3 L} dx$$

[illegible]

$$dx = P \frac{d\alpha}{\sin^2\alpha} \qquad \operatorname{tg}\alpha = \frac{P}{x} \Rightarrow$$

$$x = \frac{P}{\operatorname{tg}\alpha}$$

$$\sin\alpha = \frac{R}{r} \Rightarrow r = \frac{P}{\sin\alpha}$$

[illegible] dx [illegible]

$$dB = \frac{\mu_0 I}{2} \frac{N}{L} \sin\alpha\, d\alpha$$

[illegible]

$$B = \frac{\mu_0 I}{2} \frac{N}{L} \int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

$$B = \frac{\mu_0 I}{2} \frac{N}{L} \left[\cos\alpha_1 - \cos\alpha_2\right]$$

[illegible]

$$B = \frac{\mu_0 I}{L} N = \mu_0 n I$$

[illegible]

$$\frac{1}{2}\left(\frac{d^2}{dx^2} - x^2\right)$$

$$\left(\frac{d^2}{dx^2} - x^2\right) e^{-\frac{x^2}{2}} = \frac{d^2}{dx^2} e^{-\frac{x^2}{2}} - x^2 e^{-\frac{x^2}{2}}$$

$$= \frac{d}{dx}\left(-x e^{-\frac{x^2}{2}}\right) - x^2 e^{-\frac{x^2}{2}}$$

$$e^{-\frac{x^2}{2}} \quad e^{-\frac{x^2}{2}}$$



ملاحظات مختلفة

[illegible] الشرط [illegible]

$V = \frac{P}{m}$

$$\frac{1}{i\hbar}\left[\hat{x}\,\hat{H}\right] = \hat{V}$$

حيث P هو مؤثر [illegible] و $\hat{H}$ هو المؤثر الهاملتوني [illegible]

[illegible] التالية

$$[\hat{x}, \hat{H}] = \hat{x}\hat{H}\psi - \hat{H}\hat{x}\psi$$

$$-\frac{\hbar^2 x}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}Kx^3\psi$$

$$-\left[-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}Kx^3\psi\right]$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}Kx^3\psi$$

$$+\frac{\hbar^2}{2m}\left(x\frac{\partial^2\psi}{\partial x^2} + 2\frac{\partial\psi}{\partial x}\right) - \frac{1}{2}Kx\psi$$

$$[\hat{x}\,\hat{H}]\psi = \frac{\hbar^2}{m}\frac{d\psi}{dx}$$

$$\frac{1}{i\hbar}\left[\hat{x}\,\hat{H}\right]\psi = \frac{1}{m}\,i\hbar\frac{\partial\psi}{\partial x} = \frac{P\psi}{m}$$

[illegible]

$$\frac{1}{\hbar}[\hat{x}, \hat{H}] = \hat{v}$$

[illegible]

$$div\,(grad\,u) = \nabla^2 u$$

$$div\,(grad\,u) = \nabla\cdot(\nabla u)$$

$$\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\left(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}\right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \nabla^2 u$$

[illegible] المؤثر الهاملتوني

$$\hat{H}\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)$$

[illegible] المؤثر $\hat{\pi}$ [illegible]

$$\hat{\pi}\hat{H}\psi(x) = -\frac{\hbar^2}{2m}\hat{\pi}\frac{d^2\psi(x)}{dx^2} + \pi V(x)\psi(x)$$

$$= -\frac{\hbar^2}{2m}\frac{d^2\psi(-x)}{dx^2} + V(-x)\psi(-x)$$

$$= -\frac{\hbar^2}{2m}\frac{d^2\psi(-x)}{dx^2} + V(x)\psi(-x) = \hat{H}\hat{\pi}\psi(x)$$

[illegible]

$$(\hat{\pi}\hat{H} - \hat{H}\hat{\pi})\psi(x) = 0$$

$$[\hat{H}, \hat{\pi}] = 0$$

أهم [illegible]

[illegible] NR [illegible] الجزيئية [illegible] في الحالة [illegible] الحالات [illegible] الجزيئية [illegible] n [illegible] ε [illegible]

$$w(\varepsilon, n) = \frac{g(\varepsilon, n)\,e^{-\frac{\mu_n - \varepsilon}{kT}}}{\sum_\varepsilon\sum_n g(\varepsilon, n)\,e^{\frac{\mu_n - \varepsilon}{kT}}}$$

[illegible] $\varepsilon = n\varepsilon_0$ [illegible] n [illegible]

[illegible] n [illegible] الجزيئية [illegible] n [illegible]

$$g(\varepsilon, n) = g(n) = 1$$

$$w(\varepsilon, n) = w(n) = \frac{e^{\frac{\mu - \varepsilon}{kT}n}}{\sum e^{\frac{\mu - \varepsilon}{kT}n}}$$

[illegible]

$$\phi = \sum e^{\frac{\mu - \varepsilon}{kT}n}$$

[illegible]

$$\bar{n} = kT\frac{\partial}{\partial \mu}\ln\sum e^{\frac{\mu - \varepsilon}{kT}n}$$

[illegible] $X = e^{\frac{\mu - \varepsilon_0}{kT}}$

$$\bar{n} = kT\frac{\partial}{\partial \mu}\ln\sum_n x^n$$

[illegible]

$$\bar{n} = kT\frac{\partial}{\partial \mu}\ln\sum_{n=0}^{1}x^n = kT\frac{\partial}{\partial \mu}\ln\left(1 + e^{\frac{\mu - \varepsilon}{kT}}\right)$$

[illegible]

$$\bar{n} = \frac{1}{e^{\frac{\varepsilon_0 - \mu}{kT}} + 1}$$



ملاحظات مختلفة

اضافة جديدة (1)

معادلة شرودنغر

بيّن دي برولي أنه يمكن أن نكتب للجسيم الذي كتلته m والذي يتحرك بسرعة v طول موجة يعطى بالعلاقة

$$\lambda = \frac{h}{mv}$$

h: ثابت بلانك

$p = mv$ هو الاندفاع الخطي للجسيم

$$\lambda = \frac{h}{p}$$

معادلة الموجة المستقرة:

$$\lambda = \frac{2L}{n}$$

λ: طول الموجة

L: طول الوتر

n: عدد الحلقات

معادلة الموجة المستقرة بالشكل:

(1) $$\psi = A \sin\frac{2\pi x}{\lambda} \cos 2\pi f t$$

A: ثابت هو سعة الموجة

ψ: دالة الموجة

بالاشتقاق مرتين بالنسبة لـ x:

(2) $$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2} A \sin\frac{2\pi x}{\lambda} \cos 2\pi f t$$

أي:

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

أو:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

$$\lambda = \frac{h}{p}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2 p^2\psi}{h^2} = 0$$

الطاقة الكلية تعطى بالعلاقة:

$$E = \frac{p^2}{2m} + U$$

$$p^2 = (E - U)2m$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2\psi}{h^2}(E - U)2m = 0$$

$$h = 2\pi\hbar \Leftarrow \hbar = \frac{h}{2\pi}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2\psi}{\hbar^2}(E - U)m = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2\psi}{\hbar^2}(E - U)m = 0$$

معادلات ماكسويل

$$\vec\nabla \times \vec B = \mu_0 \vec J$$

$$\vec\nabla \times \vec E = 0$$

من قانون فاراداي للتحريض الكهرطيسي فإن القوة المحركة الكهربائية المتحرضة تعطى

$$\mathcal{E} = -\frac{d\psi}{dt}$$

كذلك القوة المحركة

$$\mathcal{E} = \oint \vec E\, d\vec l$$

حيث:

$$\psi = \int \vec B\, d\vec s$$

$$\mathcal{E} = -\frac{\partial}{\partial t}\int \vec B\, d\vec s = -\int \frac{\partial \vec B}{\partial t} d\vec s$$

$$\oint \vec E\, d\vec l = -\int \frac{\partial \vec B}{\partial t} d\vec s$$

باستخدام مبرهنة ستوكس

$$\oint \vec E\, d\vec l = \int \vec\nabla \times \vec E\, d\vec s = -\int \frac{\partial \vec B}{\partial t} d\vec s$$

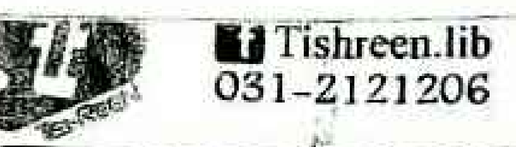
ومنه

$$\vec\nabla \times \vec E = -\frac{\partial \vec B}{\partial t}$$

$$\oint \vec B\, d\vec l = \mu_0 I = \mu_0 \int \vec J\, d\vec s$$

$$\oint \vec B\, d\vec l = \int (\vec\nabla \times \vec B)\, d\vec s = \mu_0 \int \vec J\, d\vec s$$

$$\vec\nabla \times \vec B = \mu_0 \vec J$$



ملاحظات مختلفة ←

(2) الإجابة

الجواب الثاني:

من قانون أمبير في صيغته التكاملية:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \int \vec{J}\cdot ds$$

ومن ستوكس

$$\oint \vec{B}\cdot d\vec{l} = \oint (\vec{\nabla}\times\vec{B})\cdot ds = \mu_0 \int J\, ds$$

$$\Rightarrow \nabla\times B = \mu_0 J \Rightarrow J = \frac{1}{\mu_0}(\nabla\times B)$$

$$\Rightarrow \nabla\cdot J = \frac{1}{\mu_0}\nabla\cdot(\nabla\times B) = 0$$

(2) ومن معادلة حفظ الشحنة:

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot J \Rightarrow \frac{\partial \rho}{\partial t} = 0$$

افترض ماكسويل إضافة حد إلى قانون أمبير بالشكل:

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \vec{X}$$

$$\nabla\cdot J = \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B} - \vec{X})$$

$$= -\frac{1}{\mu_0}\nabla\cdot X = -\frac{\partial \rho_f}{\partial t}$$

ومن

$$\frac{\partial \rho_f}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = \nabla\cdot\frac{\partial D}{\partial t}$$

حيث D هو متجه الإزاحة الكهربائي

$$\vec{\nabla}\cdot\left(\frac{\partial D}{\partial t}\right) = \frac{1}{\mu_0}\nabla\cdot X \Rightarrow X = \mu_0\frac{\partial D}{\partial t}$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial D}{\partial t}\right)$$

$$\nabla\times H = J + \frac{\partial D}{\partial t}$$

حيث $\vec{B} = \mu_0\vec{H}$

وهي معادلة ماكسويل الرابعة

معادلة ماكسويل الثالثة هي:

$$\vec{\nabla}\cdot\vec{B} = 0$$

المعادلة الأساسية الأولى هي:

$$\vec{\nabla}\cdot D = \rho_f$$

ρ_f هي كثافة الشحنات الحرة في واحدة الحجوم

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \qquad (a)$$

$$\vec{\nabla}\times\vec{H} = J + \frac{\partial D}{\partial t} \qquad (b)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad (c)$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f \qquad (d)$$

مع العلاقات

$$\vec{B} = \mu\vec{H}, \quad \vec{D} = \varepsilon\vec{E}$$

ε، μ ثابتا الوسط

معادلات الحقل في الفراغ بدلالة الكمون السلمي ϕ والكمون المتجهي $\vec{A}$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

ويمكن أن نعرف $\vec{E}$ بالشكل:

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

وإذا كان $\vec{B}$ ثابتاً فإن $\frac{\partial \vec{A}}{\partial t} = 0$

$$\vec{E} = -\nabla\phi$$

$$\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times(\nabla\phi) - \nabla\times\frac{\partial \vec{A}}{\partial t}$$

ولكن

$$\vec{\nabla}\times\frac{\partial A}{\partial t} = \frac{\partial}{\partial t}\nabla\times\vec{A}$$

$$= \frac{\partial B}{\partial t} \quad \text{لأن } \nabla\times(\nabla\phi) = 0$$

فالمعادلة تأخذ الشكل:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$$

وهي معادلة ماكسويل الأولى

ملاحظات مختلفة ←

[illegible]

[illegible]

$$\hat{A}\,\psi_i(x) = a_i\,\psi_i(x) \quad (1)$$

$$\hat{A}^*\,\psi^*(x) = a_i^*\,\psi_i^*(x) \quad (2)$$

[illegible]

$$\int_{-\infty}^{+\infty} \psi^* \hat{A}\,\psi_i\,dx = \int_{-\infty}^{+\infty} a_i\,\psi_i^*\,\psi_i\,dx$$

$$\int_{-\infty}^{+\infty} \hat{A}^*\,\psi_i^*\,\psi_i\,dx = \int_{-\infty}^{+\infty} a_i^*\,\psi_i^*\,\psi_i\,dx$$

[illegible]

$$\int_{-\infty}^{+\infty} a_i\,\psi^*\,\psi\,dx - \int a_i^*\,\psi_i^*\,\psi_i\,dx$$

$$\Rightarrow (a_i - a_i^*) \int \psi^*\,\psi\,dx = 0$$

[illegible] a_i [illegible] a_i^* [illegible]

[illegible]

$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,dz}{r^2}\,\vec{u}$$

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\,dl}{r^2}\,\vec{u}$$

[illegible] dE [illegible]

$$dE = dE'\cos\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,dz}{r^2}\cos\theta$$

[illegible]

$$E = \int dE = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{dz}{r^2}\cos\theta$$

[illegible]

$$r = \frac{a}{\cos\theta} \quad , \quad z = a\,tg\,\theta$$

$$dz = \frac{a}{\cos^2\theta}\,d\theta$$

[illegible]

$$E = \frac{\lambda}{4\pi\varepsilon_0 a} \int_{\theta_1}^{\theta_2} \cos\theta\,d\theta$$

[illegible]

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}\,\sin\theta_1$$

[illegible]

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

[illegible]

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0$$

[illegible]

$$\operatorname{div}(\vec{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

[illegible]

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{s} \left\{ \frac{\partial}{\partial q_i}(\rho\,\dot{q}_i) + \frac{\partial}{\partial p_i}(\rho\,\dot{p}_i) \right\} = 0 \quad (1)$$

[illegible] $\frac{\partial \rho}{\partial t}$ [illegible]

$$\sum_{i=1}^{s} \left\{ \frac{\partial}{\partial q_i}(\rho\,\dot{q}_i) + \frac{\partial}{\partial p_i}(\rho\,\dot{p}_i) \right\} = 0 \quad (3)$$

[illegible]

$$\sum_{i=1}^{s} \left(\dot{q}_i\,\frac{\partial \rho}{\partial q_i} + \dot{p}_i\,\frac{\partial \rho}{\partial p_i} \right) + \rho \sum \left(\frac{\partial \dot{q}_i}{\partial q_i} + \frac{\partial \dot{p}_i}{\partial p_i} \right) = 0 \quad (4)$$

[illegible]

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad , \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

[illegible]

$$\frac{\partial \dot{q}_i}{\partial q_i} = \frac{\partial^2 H}{\partial q_i\,\partial p_i} \quad , \quad \frac{\partial \dot{p}_i}{\partial p_i} = -\frac{\partial^2 H}{\partial q_i\,\partial p_i}$$

[illegible]

$$\frac{\partial \dot{q}_i}{\partial q_i} = -\frac{\partial \dot{p}_i}{\partial p_i}$$



ملاحظات مختلفة ←

مثال: $\vec{A} = x^3 z^2 \vec{i} - 3xy^2 z^3 \vec{j} + 2x^2 y^2 z^2 \vec{k}$
أوجد قيمة $\operatorname{div}\vec{A}$ وكذلك $\operatorname{rot}\vec{A}$
في النقطة $P(1,1,1)$

الحل:

$$\operatorname{div}\vec{A} = \vec{\nabla}\cdot\vec{A}$$

$$\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(x^3 z^2 \vec{i} - 3xy^2 z^3 \vec{j} + 2x^2 y^2 z^2 \vec{k}\right)$$

$$= \frac{\partial}{\partial x}(x^3 z^2) + \frac{\partial}{\partial y}(-3xy^2 z^3) + \frac{\partial}{\partial z}(2x^2 y^2 z^2)$$

$$\vec{\nabla}\cdot\vec{A} = 3x^2 z^2 - 6xyz^3 + 4x^2 y^2 z$$

عند النقطة $P(1,1,1)$ ينتج

$$\vec{\nabla}\cdot\vec{A} = 3 - 6 + 4 = 1$$

من تعريف [illegible]

$$\operatorname{rot}\vec{A} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 z^2 & 3xy^2 z^3 & 2x^2 y^2 z^2 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{A} = \left[\frac{\partial(2x^2y^2z^2)}{\partial y} - \frac{\partial(-3xy^2z^3)}{\partial z}\right]\vec{i} + \left[\frac{\partial(x^3z^2)}{\partial z} - \frac{\partial(2x^2y^2z^2)}{\partial x}\right]\vec{j} + \left[\frac{\partial(-3xy^2z^3)}{\partial x} - \frac{\partial(x^3z^2)}{\partial y}\right]\vec{k}$$

$$= (4x^2yz^2 + 9xy^2z^2)\vec{i} + (2x^3z - 4xy^2z^2)\vec{j} + (-3y^2z^3)\vec{k}$$

في النقطة $P(1,1,1)$

$$\vec{\nabla}\times\vec{A} = 13\vec{i} - 2\vec{j} - 3\vec{k}$$

أوجد $\overrightarrow{\operatorname{grad}} V$ في النقطة $P(1,1,1)$

$$V(x,y,z) = 2y^2 z - x^3 y^2$$

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}$$

$$\vec{\nabla}V = \frac{\partial(2y^2z - x^3y^2)}{\partial x}\vec{i} + \frac{\partial(2y^2z - x^3y^2)}{\partial y}\vec{j} + \frac{\partial(2y^2z - x^3y^2)}{\partial z}\vec{k}$$

$$\vec{\nabla}V = (-3x^2y^2)\vec{i} + (4yz - 2x^3y)\vec{j} + (2y^2)\vec{k}$$

$$\vec{\nabla}V = -3\vec{i} + 2\vec{j} + 2\vec{k}$$

يعطى أن القوة المغناطيسية $\vec{F}$ التي تؤثر بها حقل التحريض المغناطيسي على شحنة q تتحرك بالسرعة $\vec{V}$ تعطى بالعلاقة التالية:

$$\vec{F} = q(\vec{V}\times\vec{B})$$

مثال:
إذا كانت (dq) شحنة عنصرية تتحرك بسرعة $\vec{V}$ في حقل تحريض مغناطيسي $\vec{B}$ فيحسب القوة المغناطيسية سواء أكانت الشحنة موجبة أم سالبة فإن القوة عمودية على كل من اتجاه الحقل التحريض المغناطيسي $\vec{B}$ وعلى اتجاه حركة الشحنة المتحركة بسرعة [illegible]

$$d\vec{F} = I(d\vec{l}\times\vec{B})$$

أي عنصر من التيار [illegible] ... الشحنة العنصرية هي: $d\vec{l} = \vec{V}\,dt$
وبالتعويض:

$$d\vec{F} = I(\vec{V}\times\vec{B})\,dt$$

ولكن $dq = I\,dt$ بالتعويض نجد:

$$d\vec{F} = dq(\vec{V}\times\vec{B})$$

وبالتالي تكون القوة المغناطيسية $\vec{F}$ التي يخضع لها حقل التحريض المغناطيسي على الشحنة النقطية q ذات السرعة $\vec{V}$ هي:

$$\vec{F} = q(\vec{V}\times\vec{B})$$

إذا كانت $\vec{A}$ دالة متجهة مستمرة وكذلك مشتقاتها أثبت أن $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

الحل: بفرض أن $\vec{A}$ دالة متجهة وكذلك مشتقاتها

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

من خواص المحددات نستطيع الحصول على أن المحدد يساوي الصفر



ملاحظات مختلفة

[illegible]

[illegible]

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

[illegible]

$$V_b - V_a = \int_a^b dV$$

[illegible]

$$dV = -E\,dl = -E\,dl\cos\theta$$

$$= -E_l\,dl$$

[illegible]

$$E_l = E\cos\theta$$

[illegible]

$$\frac{dV}{dl} = -E_l$$

[illegible] (X, Y, Z) [illegible]

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

[illegible]

(1) [illegible]

$$dV = -\vec{E} \cdot d\vec{l} = -E\,dl$$

[illegible]

$$E = -\left(\frac{dV}{dl}\right)_{max}$$

[illegible]

$$\vec{E} = -\text{grad}\,V$$

[illegible]

$$(\text{const}) \Rightarrow V = \text{Cost}$$

[illegible]

$$\vec{\nabla}(U \cdot V) = U\vec{\nabla}V + V\vec{\nabla}U$$

[illegible]

$$\vec{\nabla}(U \cdot V) = \vec{i}\frac{\partial}{\partial x}(U \cdot V) + \vec{j}\frac{\partial}{\partial y}(UV) + \vec{k}\frac{\partial}{\partial z}(U \cdot V)$$

$$\vec{\nabla}(U \cdot V) = \vec{i}\left(\frac{\partial U}{\partial x}V + U\frac{\partial V}{\partial x}\right) + \vec{j}\left(\frac{\partial U}{\partial y}V + U\frac{\partial V}{\partial y}\right) + \vec{k}\left(\frac{\partial U}{\partial z}V + U\frac{\partial V}{\partial z}\right)$$

$$\vec{\nabla}(U \cdot V) = U\left(\vec{i}\frac{\partial V}{\partial x} + \vec{j}\frac{\partial V}{\partial y} + \vec{k}\frac{\partial V}{\partial z}\right) + V\left(\vec{i}\frac{\partial U}{\partial x} + \vec{j}\frac{\partial U}{\partial y} + \vec{k}\frac{\partial U}{\partial z}\right)$$

[illegible] grad U, grad V

$$\vec{\nabla}(UV) = U\vec{\nabla}V + V\vec{\nabla}U$$

$$\text{div}(V\vec{A}) = V\,\text{div}\,\vec{A} + \text{grad}\,V \cdot \vec{A}$$

[illegible]

$$\text{div}(V\vec{A}) = \vec{\nabla} \cdot (V\vec{A})$$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot \left(Vx\vec{i} + Vy\vec{j} + Vz\vec{k}\right)$$

$$= \frac{\partial V(x)}{\partial x} + \frac{\partial V(y)}{\partial y} + \frac{\partial Vz}{\partial z}$$

$$= V\frac{\partial x}{\partial x} + V\frac{\partial y}{\partial y} + V\frac{\partial z}{\partial z} + x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$$

$$= V\left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) + \left(x\vec{i} + y\vec{j} + z\vec{k}\right) \cdot \left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

$$\text{div}(V\vec{A}) = V\,\text{div}\,\vec{A} + \vec{\nabla}V \cdot \vec{A}$$